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# Prima lezione: il disco di Airy e l'occhio limitato per diffrazione

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Applicata



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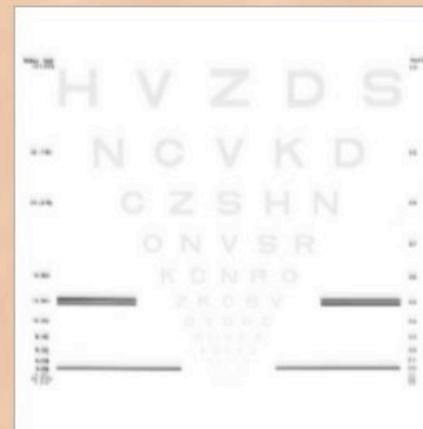
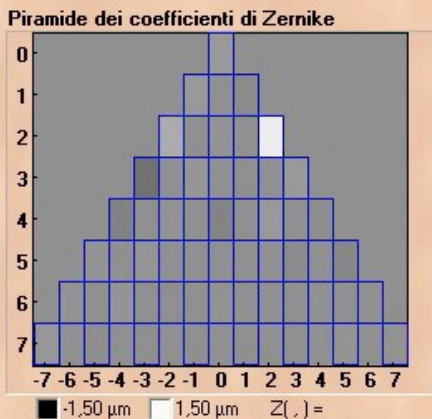
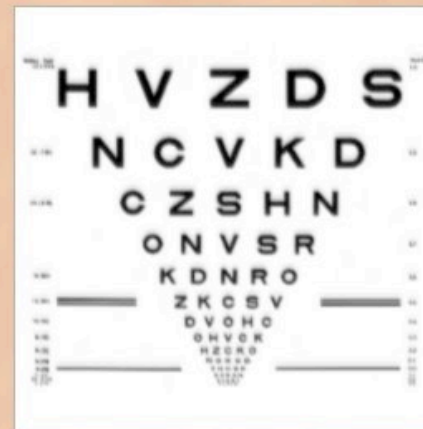
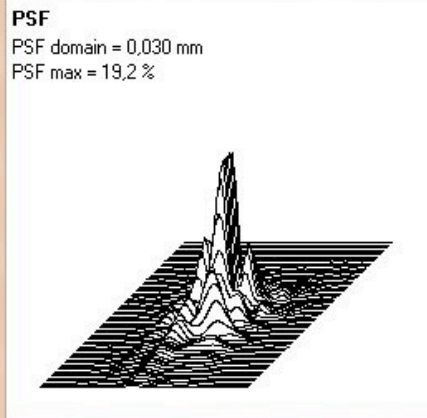
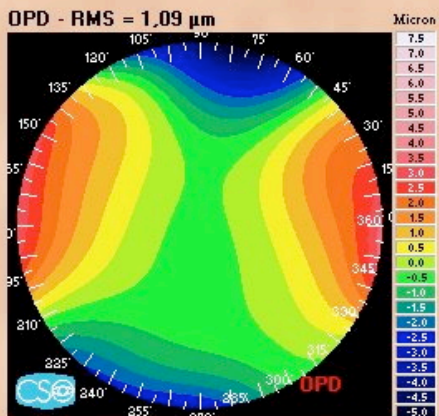
# Programma

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## Qualità dell'Immagine retinica

- Diffrazione
- Figura di Airy
- Basi matematiche: trasformata di Fourier
- Scattering
- OTF e MTF dell'occhio
- Rappresentazione delle funzioni d'onda
- Spettro di frequenza
- Applicazioni a topografi e aberrometri





Pupilla d'entrata 5mm



# Programma 2

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## Trasmittanza oculare e illuminamento retinico

- Trasmittanza dei mezzi oculari
- Diametro pupillare
- Illuminamento retinico
- Effetto Stiles Crawford
- Scattering oculare
- Polarizzazione oculare
- Radiometria e fotometria dell'occhio
- Il campionamento retinico: formula di Nyquist



## Programma 3

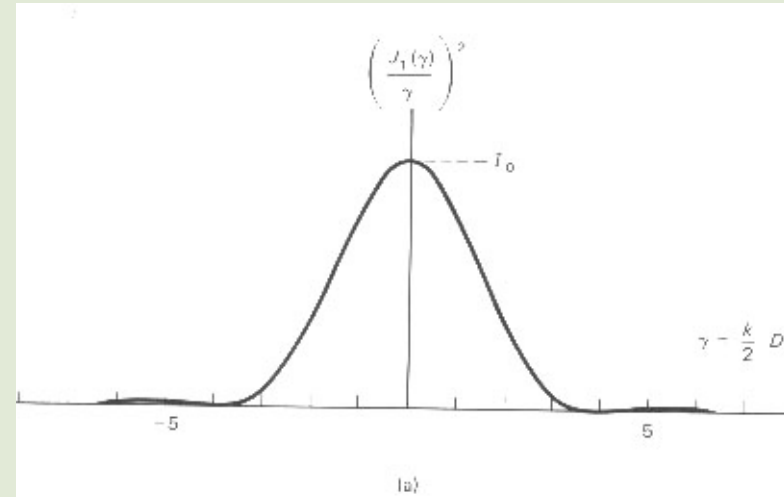
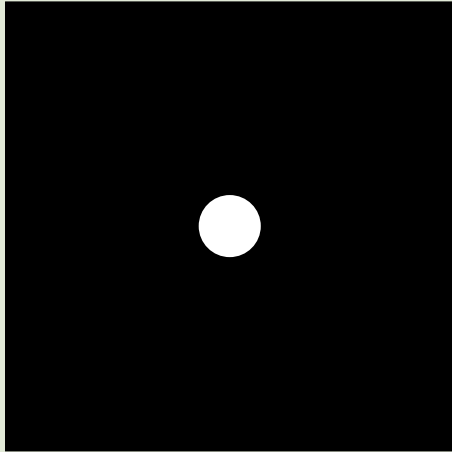
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- **Visione del colore**
- Risposta di coni e bastoncelli
- Metamerismo
- Costanza del colore
- Esperimenti per la risposta dei bastoncelli e la sensibilità oculare



# Il disco di Airy

Apertura circolare



Disco di Airy



(b)



al Sir George Biddell Airy: Inventore degli occhiali per astigmatici

## Il disco di Airy

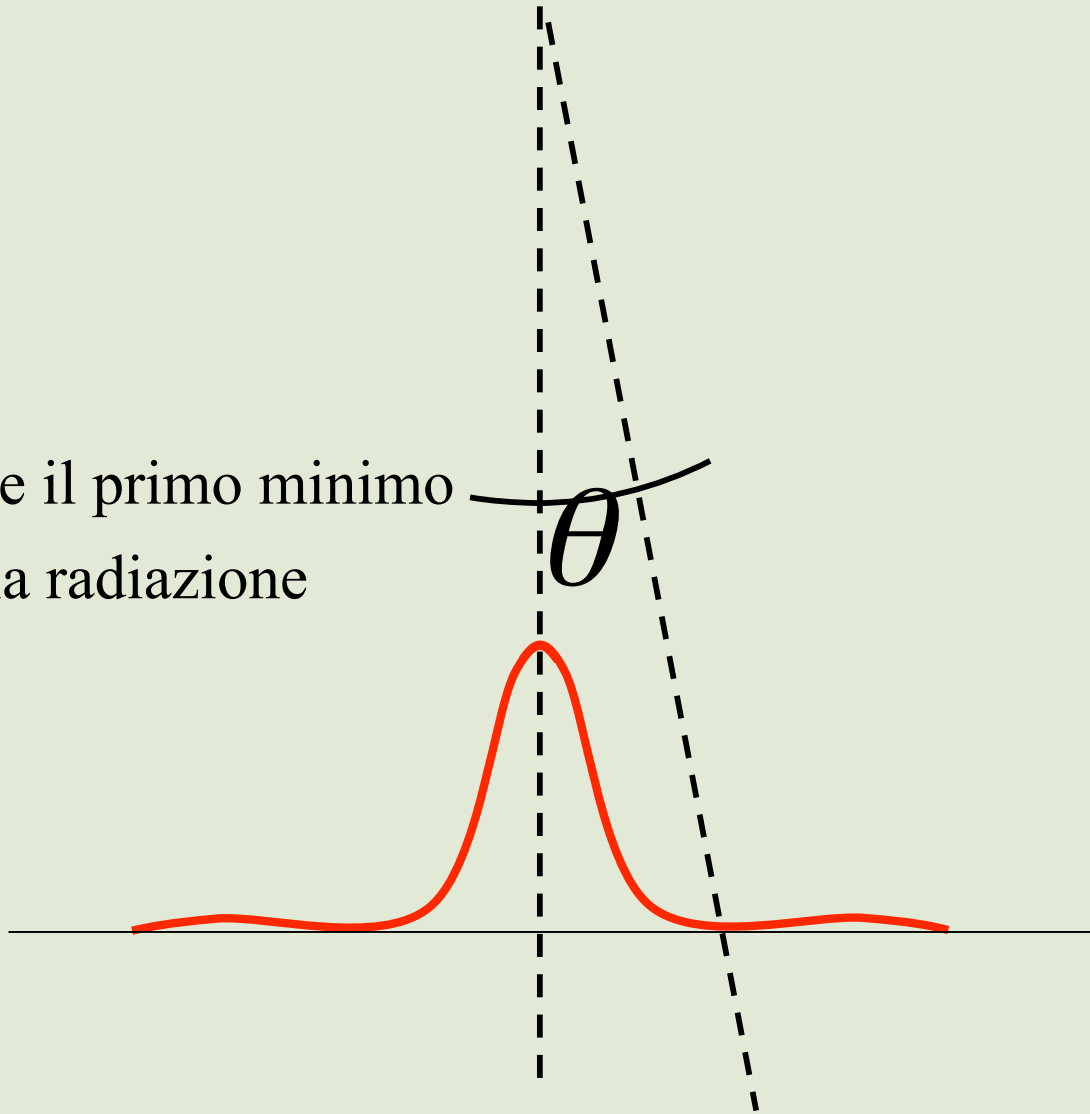
$$\theta = \frac{1.22 \cdot \lambda}{a}$$

$\theta$  = angolo tra il massimo e il primo minimo

$\lambda$  = lunghezza d'onda della radiazione

$a$  = diametro della pupilla

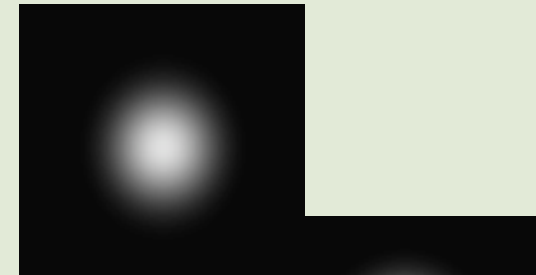
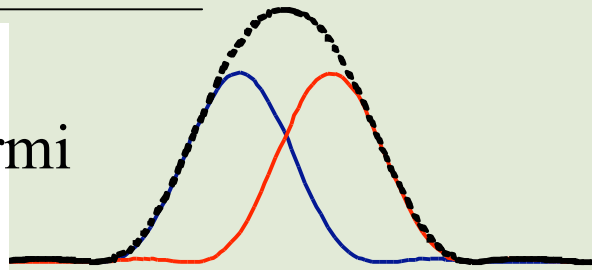
Attenzione: l'angolo è  
espresso in radianti



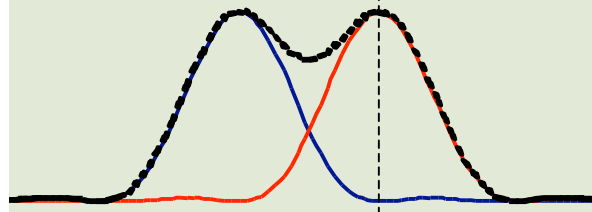
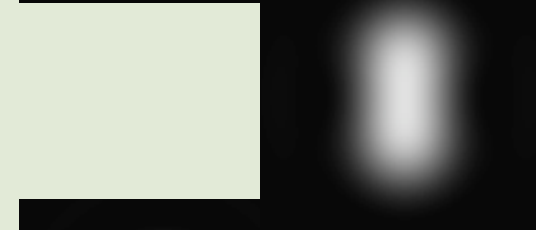
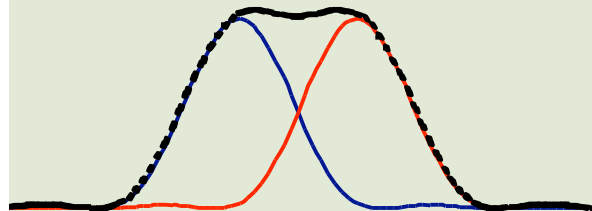


# Limite di risoluzione di Rayleigh

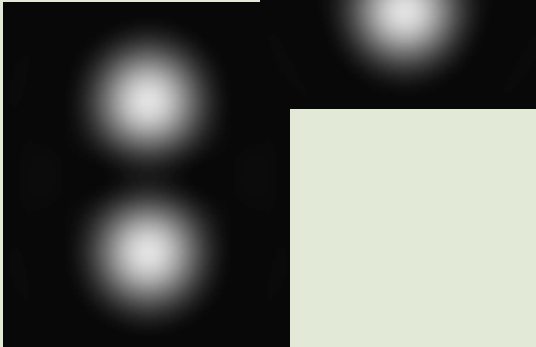
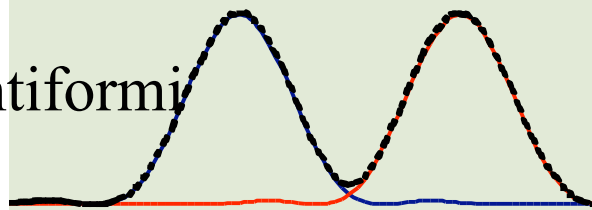
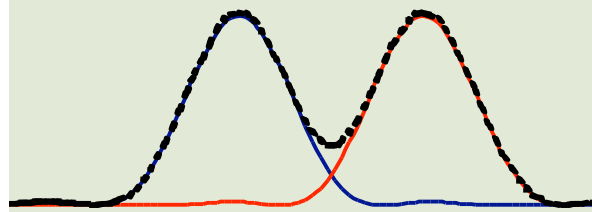
Sorgenti puntiformi non risolubili



Limite di risoluzione di Rayleigh



Sorgenti puntiformi risolte



## Esercizio

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- Calcolare la risoluzione per una pupilla di 3mm
- Fatto in `risoluzione_occhio.m`



# Point Spread Function vs. diametro pupillare

1 mm



2 mm



3 mm



4 mm



5 mm



6 mm



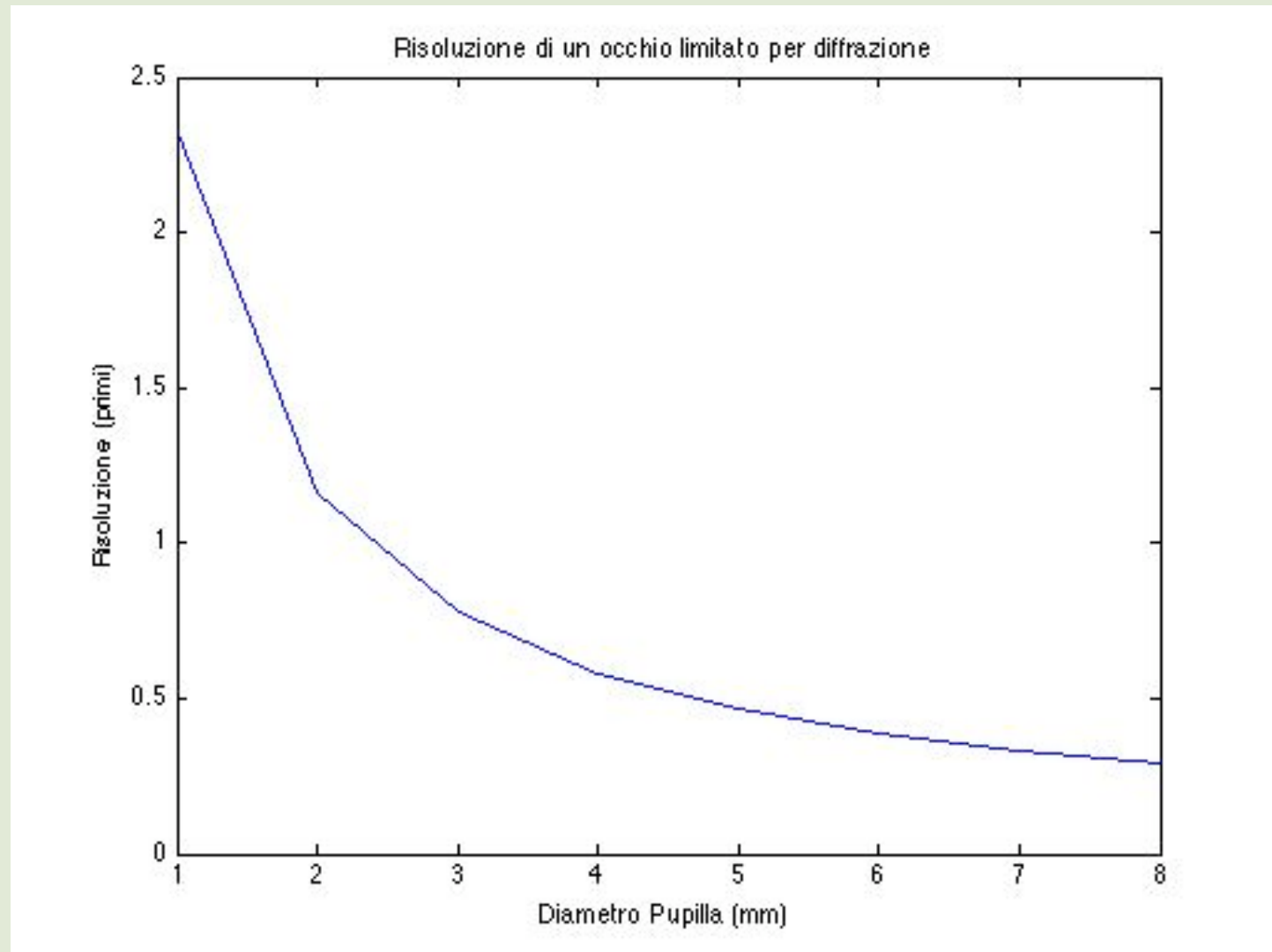
7 mm

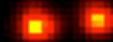
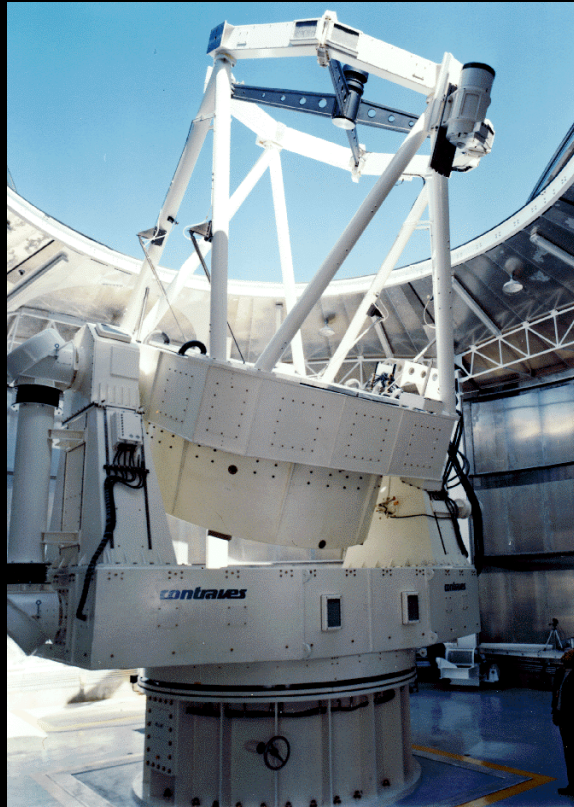


Occhio privo di aberrazioni



# Risoluzione





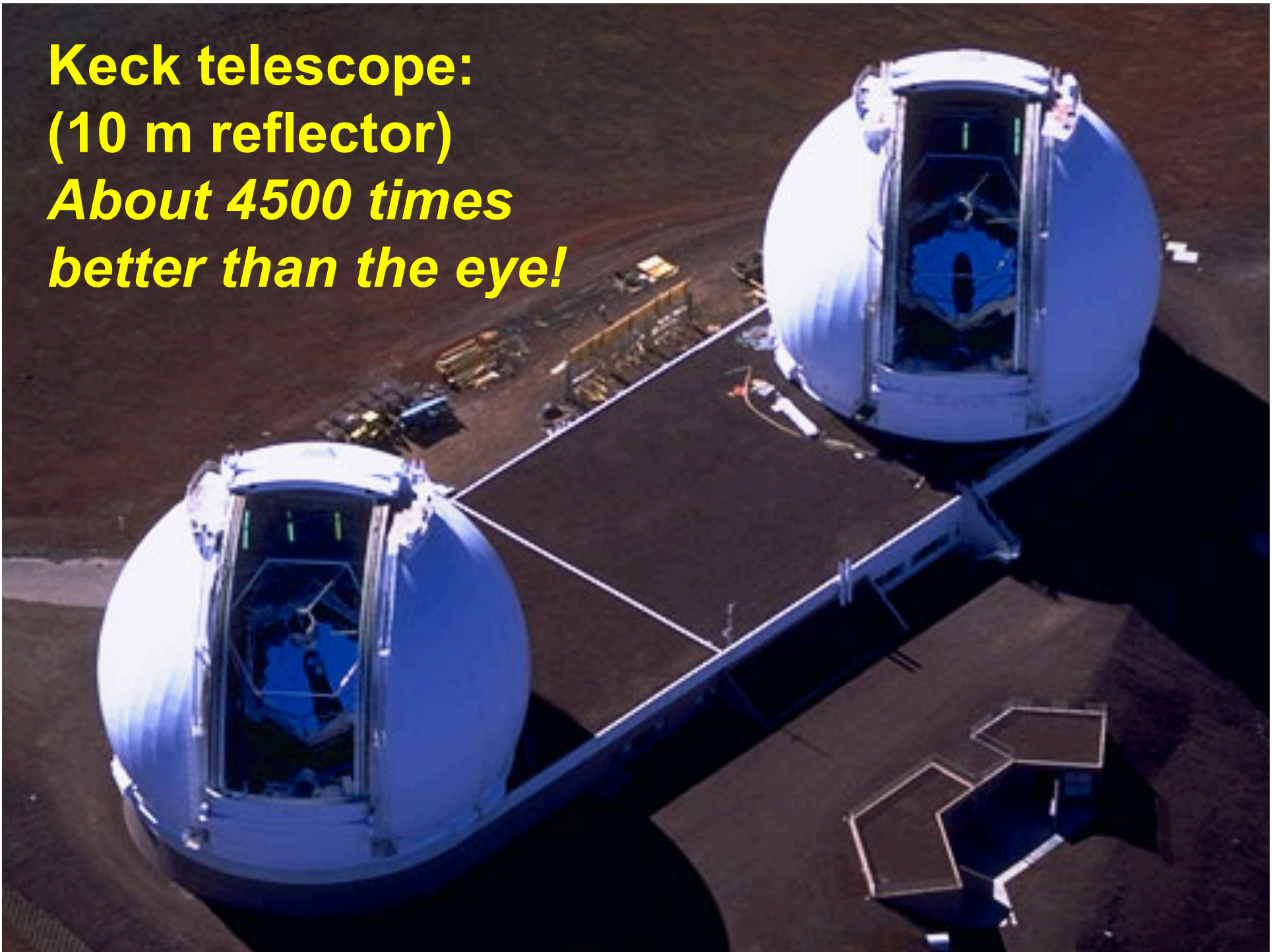
AO image of binary star k-Peg on the 3.5-m telescope at the Starfire Optical Range, Albuquerque, NM, September, 1997.

$$\theta_{\min} = \frac{1.22 \cdot \lambda}{a} = \frac{1.22 \cdot 900 \times 10^{-9}}{3.5} = 0.064 \text{ seconds of arc}$$

About 1000 times better than the eye!



**Keck telescope:  
(10 m reflector)  
*About 4500 times  
better than the eye!***



# Le dimensioni delle immagini

## Angolo visivo

L'angolo visivo è l'angolo sotteso dall'immagine retinica rispetto al secondo punto nodale. E' uguale all'angolo sotteso dall'oggetto rispetto al primo punto nodale

I punti nodali sono i punti di un sistema ottico in cui la luce entra ed esce con lo stesso angolo.

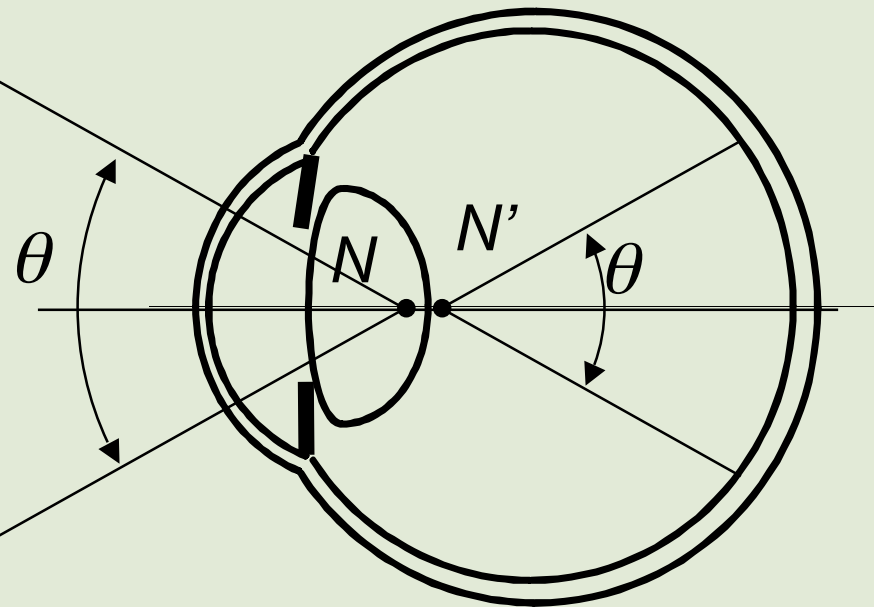
Il secondo punto nodale dell'occhio si trova a circa 16.5 mm dalla retina

Un'immagine di 1 mm sulla retina...

$$\tan \theta = \frac{0.5}{16.5} \Rightarrow \theta = 1.73^\circ$$

$$\text{visual angle} = 2\theta = 3.47^\circ$$

$$1^\circ = 288 \mu m$$



## Angoli importanti

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- 1 radiante = 57.29 gradi
- 1 grado = .0174 radianti = 17.4 mrad
- 1 primo = .29 mrad
- 1 mrad = 3.44 primi
- 1 primo = 4.8 micron (*depends on axial length*)
- 1 cono foveale = 2.5 microns (*con variazioni da soggetto a soggetto*)





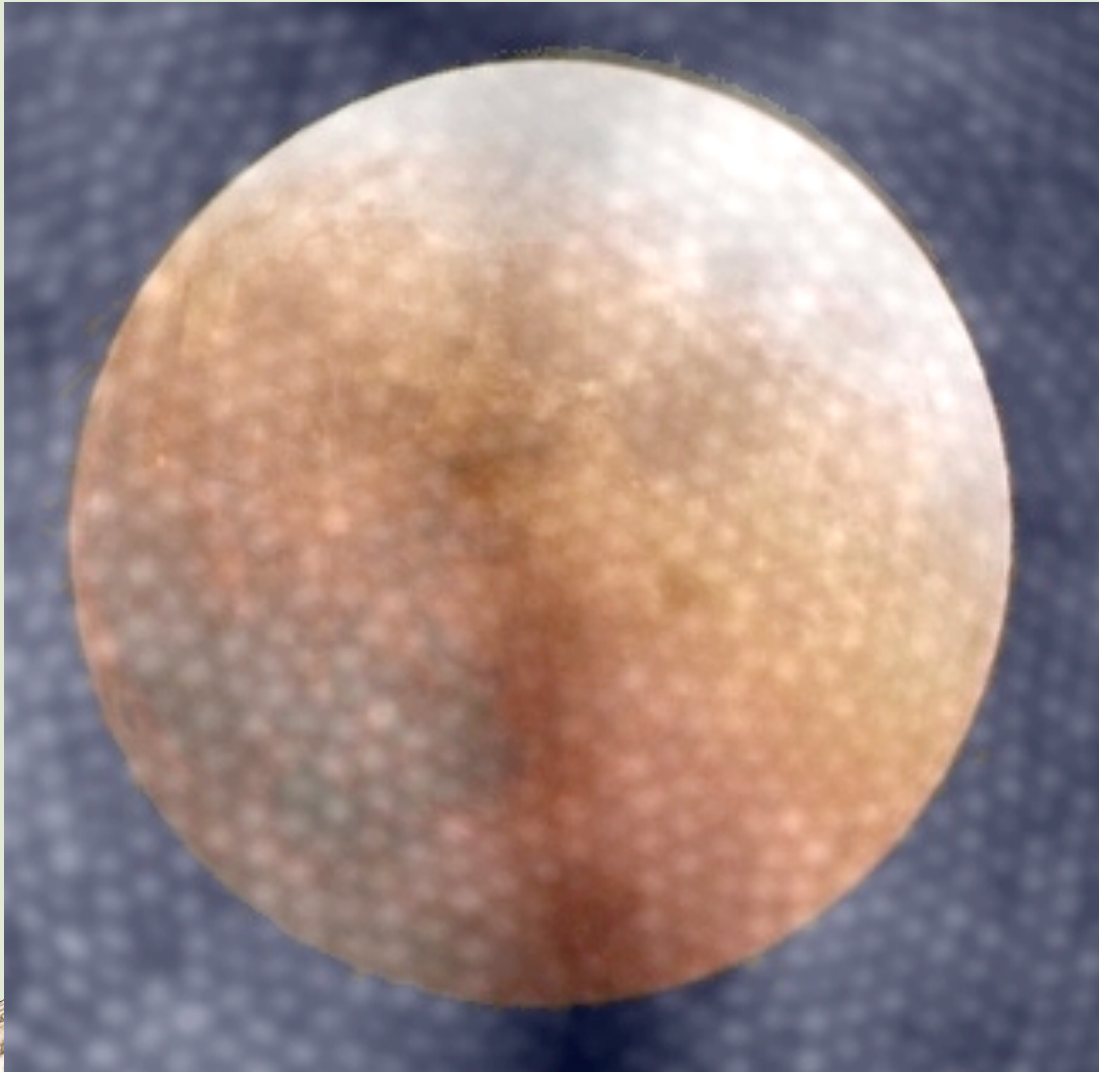
## Risoluzione

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- Dato che la distanza tra i centri di due coni nella fovea è di circa 2.5 micron, servono 5 micron per poter risolvere due punti
- 5 micron corrispondono a un primo



## Immagine di un'eclissi di luna vista sulla retina a un grado dalla fovea



La Luna sottende circa 0.5 Gradi

I coni a 1 grado dalla retina hanno un diametro di circa 5 micron.

L'immagine della Luna sulla retina è circa 144 microns

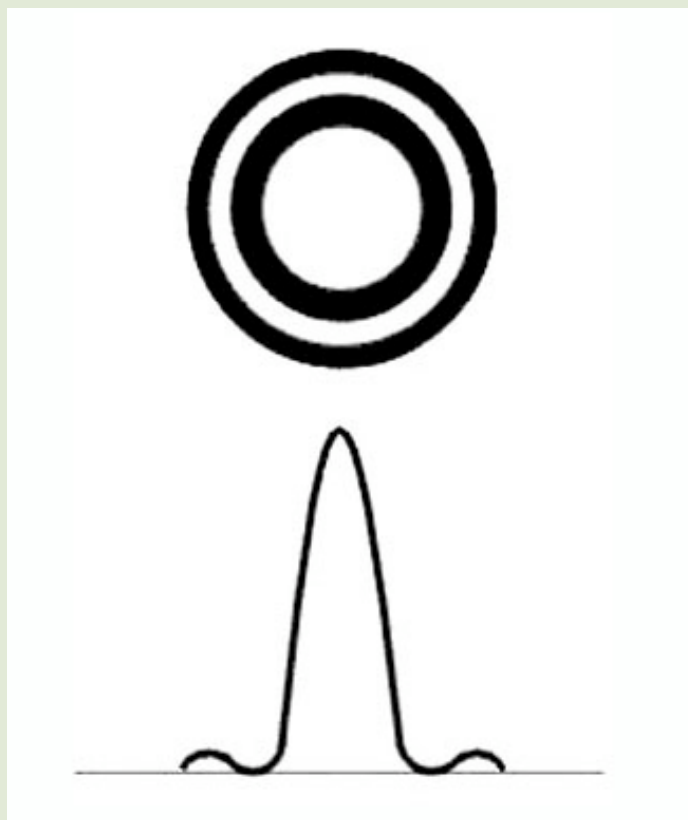
L'immagine è campionata da circa 29 cones come diametro (~650 in totale)



# Point spread function

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Il grafico dell'illuminamento dell'immagine di un punto viene chiamato Point Spread Function



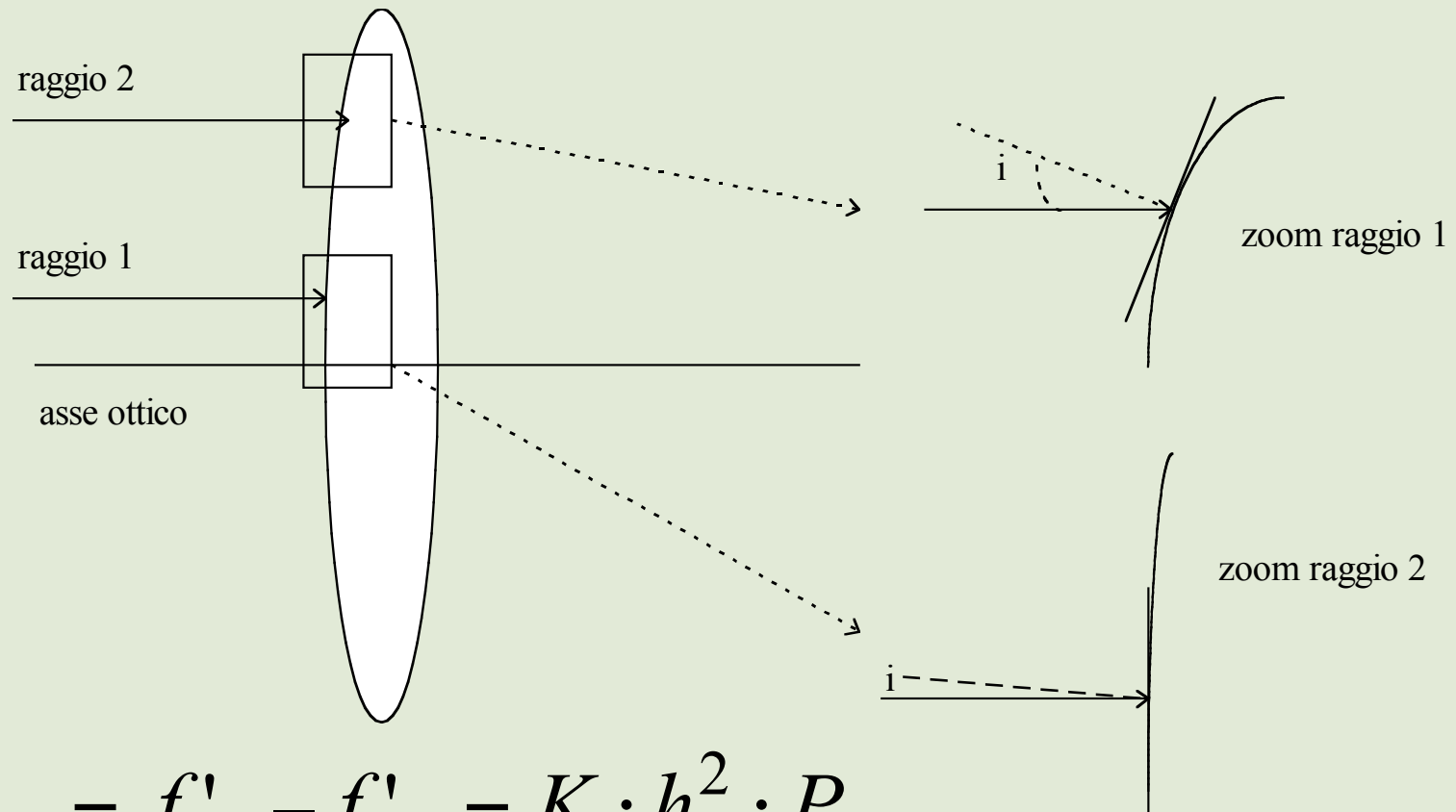
# Approssimazioni dell'ottica al primo e terzo ordine

$i$	0°	5°	10°	20°	30°	40°
$i$ (rad)	0	0.08722	0.17444	0.34888	0.52333	0.69777
$\text{Sin } i$	0	0.08711	0.17356	0.34185	0.49977	0.64251
% errore	0	0.12690	0.50898	2.05790	4.71481	8.60076
$i - \frac{i^3}{3!}$	0	0.00711	0.17355	0.34181	0.49944	0.64115
% errore	0	0.00004	0.00077	0.01256	0.06502	0.21207

$$\sin i = i - \frac{i^3}{3!} + \frac{i^5}{5!} - \frac{i^7}{7!} + \dots$$



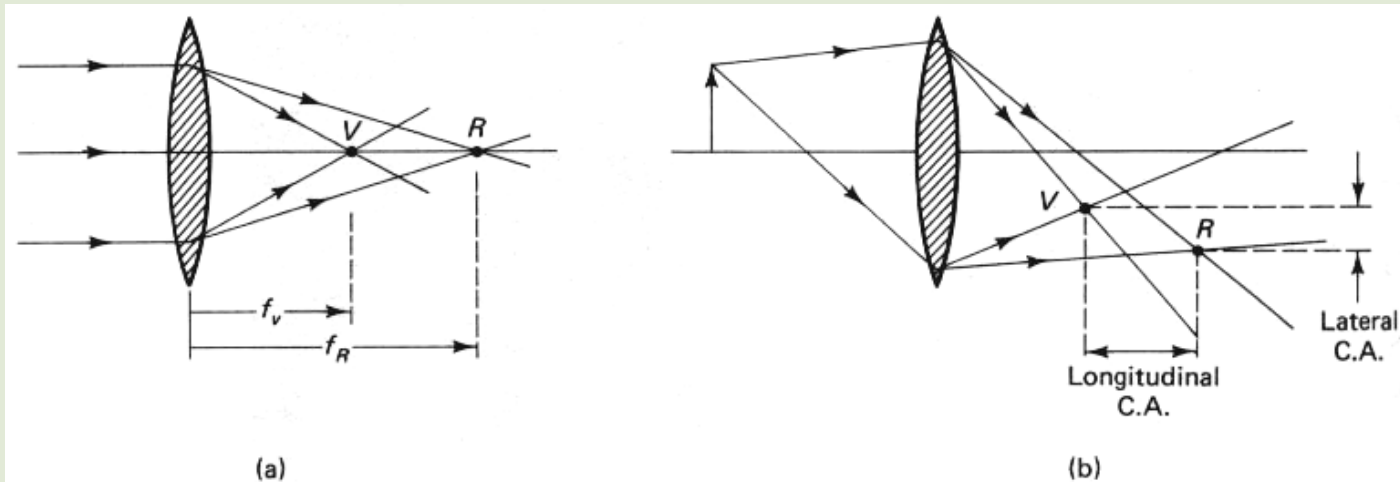
# L'aberrazione sferica



$$A_{S.L.} = f'_p - f'_m = K \cdot h^2 \cdot P$$



# Chromatic Aberration



**Figure 5-12** Chromatic aberration (exaggerated) for a thin lens, illustrating the effect on the focal length (a) and the lateral and longitudinal misses (b) for red ( $R$ ) and violet ( $V$ ) wavelengths.



# Chromatic Aberration

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Chromatic difference of refraction

$$L' = L + F$$

by definition, in the human eye

$$K' = K + F_e \quad \Rightarrow \quad K = K' - F_e$$

$$K = \frac{1}{k}$$

*refraction of the eye*

where

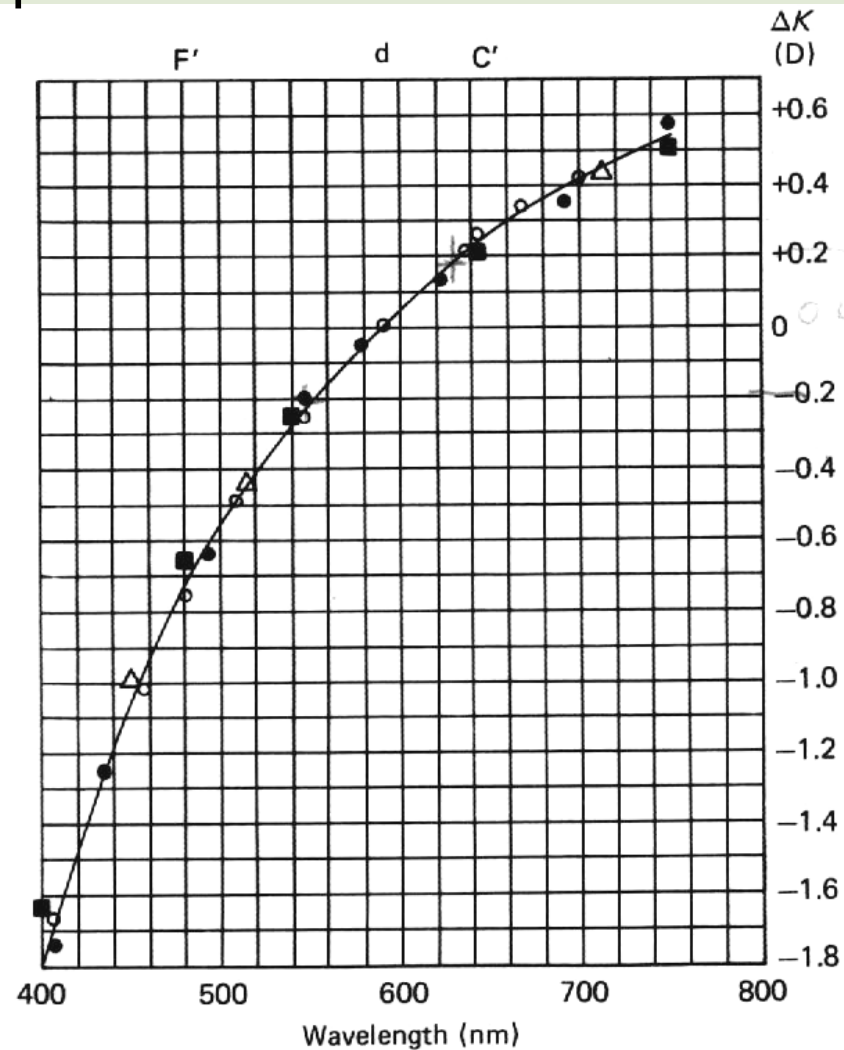
$$K' = \frac{n'}{k'}$$

$$F_e = \frac{n'}{f'}$$

*power of the eye*



# Chromatic Aberration



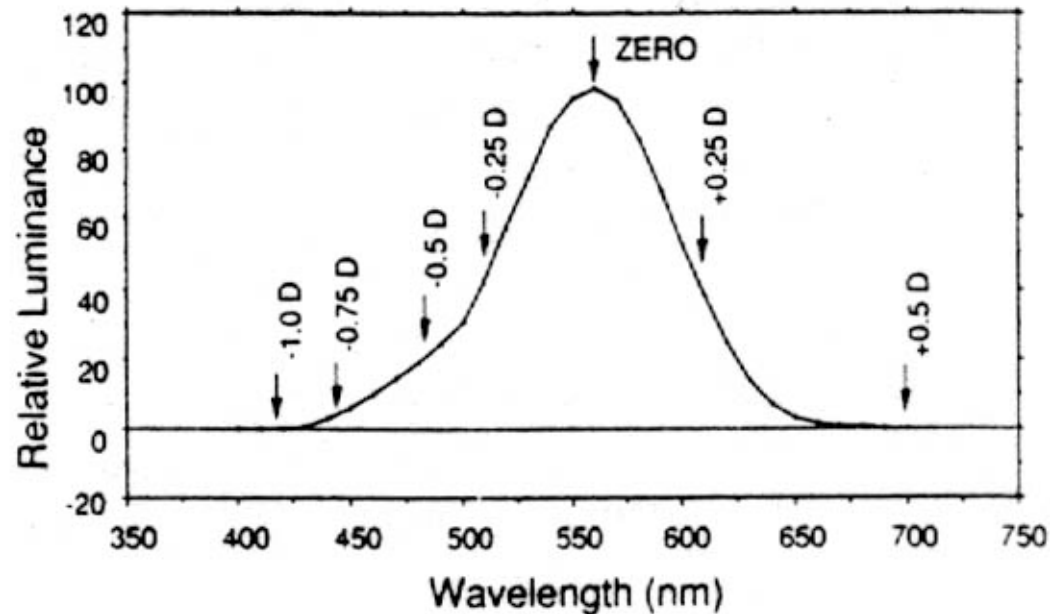
**Figure 15.5.** Chromatic difference of refraction. Experimental results adjusted for  $\lambda_0 = 587.6$  nm. Results of ● Wald and Griffin △ Ivanoff ○ Bedford and Wyszecki. The curved line represents the mean. ■ indicates calculated results for the schematic eye.

figure from Bennet and Rabbetts, 1989





# Chromatic Aberration



**Figure 3.** The significance of chromatic defocus depends on luminance. The solid curve shows the luminance spectrum of white-light emitted by the P4 phosphor of cathode ray tubes and arrows mark the amount of defocus if the eye accommodates for 550 nm. When the peak of the luminance spectrum is in focus, most of the light is less than 0.25 D out of focus.



# Aberrazioni totali dell'occhio

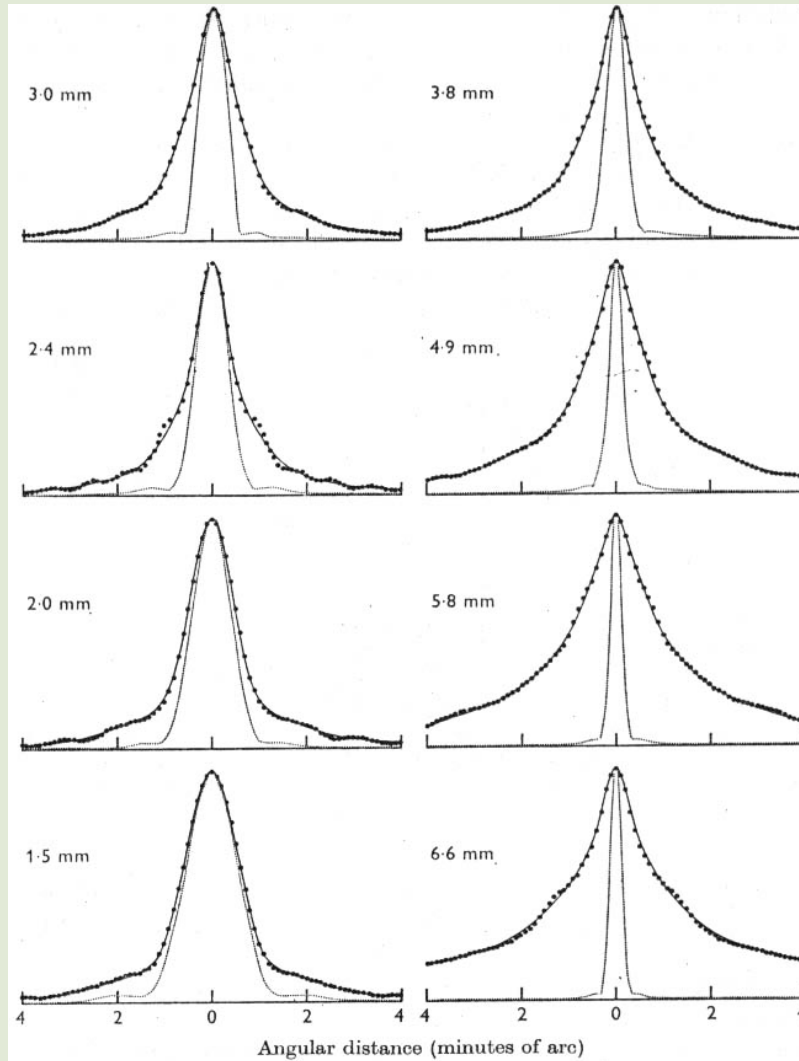


Fig. 10. Optical linespread functions of the human eye. Each curve represents the normalized distribution of illuminance occurring on the fundus for a thin line source of light. Dots occur at 0.1 min increments. Narrower curve indicates the diffraction image of a line at the given pupil diameter.



# Point Spread Function vs. Pupil Size Perfect Eye

1 mm



2 mm



3 mm



4 mm



5 mm



6 mm



7 mm



# Point Spread Function vs. Pupil Size Typical Eye

1 mm



2 mm



3 mm



4 mm



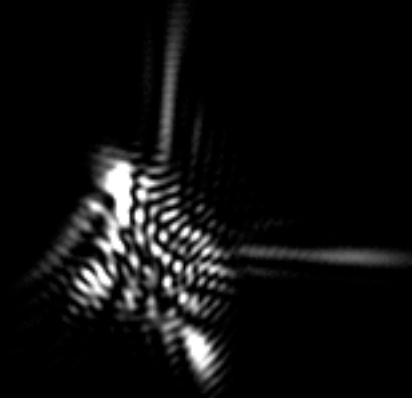
5 mm



6 mm



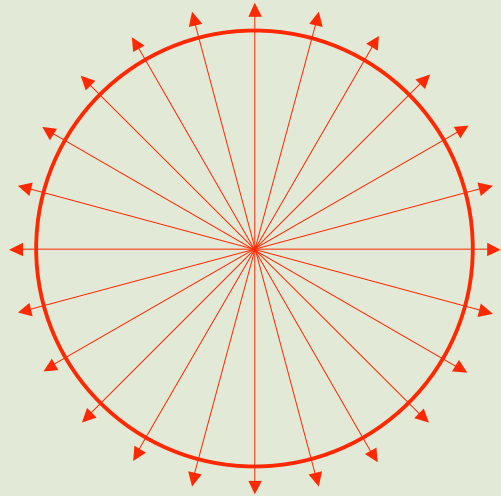
7 mm



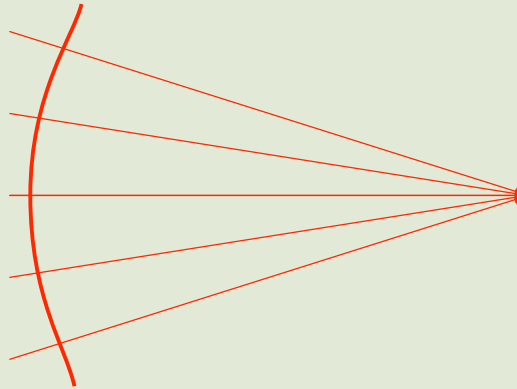
Observe your own point spread function



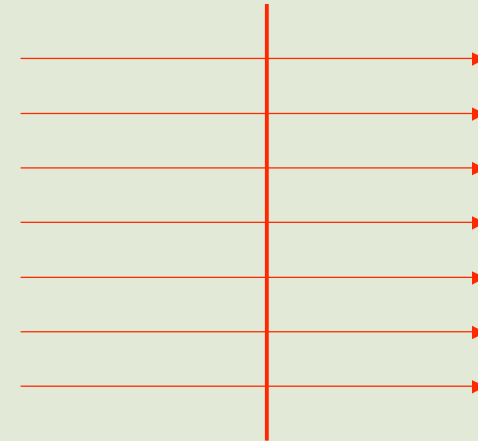
# Raggi e fronti d'onda



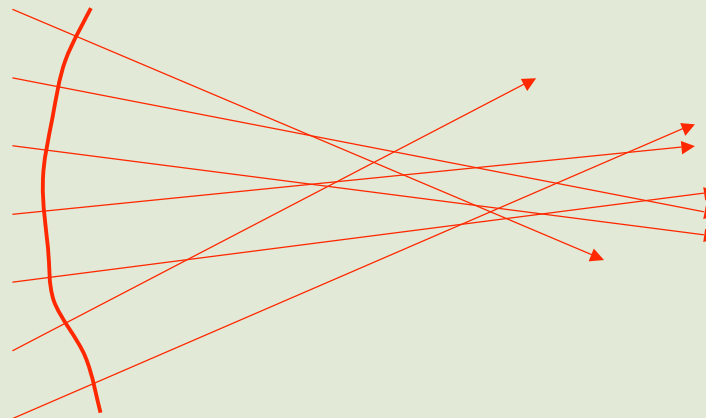
a) Spherical diverging wavefront formed by a point source



b) Spherical converging wavefront formed by a lens



c) Planar wavefront



d) Aberrated wavefront

